

Electromagnetic polarizabilities of the nucleon and properties of the σ -meson pole contribution

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Abstract. The t -channel contribution to the difference of electromagnetic polarizabilities of the nucleon, $(\alpha - \beta)^t$, can be quantitatively understood in terms of a σ -meson pole in the complex t -plane of the invariant scattering amplitude $A_1(s, t)$ with properties of the σ -meson as given by the quark-level Nambu–Jona-Lasinio model (NJL). Equivalently, this quantity may be understood in terms of a cut in the complex t -plane where the properties of the σ -meson are taken from the $\pi\pi \rightarrow \sigma \rightarrow \pi\pi$, $\gamma\gamma \rightarrow \sigma \rightarrow \pi\pi$ and $N\bar{N} \rightarrow \sigma \rightarrow \pi\pi$ reactions. This equivalence may be understood as a sum rule where the properties of the σ -meson as predicted by the NJL model are related to the $f_0(600)$ particle observed in the three reactions. In the following, we describe details of the derivation of $(\alpha - \beta)^t$ making use of predictions of the quark-level NJL model for the σ -meson mass.

PACS. 11.55.Fv Dispersion relations – 11.55.Hx Sum rules – 13.60.Fz Elastic and Compton scattering – 14.70.Bh Photons

1 Introduction

The σ -meson is an indispensable supplement of the pion [1]. In terms of nonstrange quarks, the argument is that four $q\bar{q}$ states should correspond to four mesons where the neutral members of the (π, σ) isospin quartet have the flavor structures $|\pi^0\rangle = (u\bar{u} - d\bar{d})/\sqrt{2}$ and $|\sigma\rangle = (u\bar{u} + d\bar{d})/\sqrt{2}$. According to its flavor structure the σ -meson is a scalar-isoscalar particle with relative angular momentum $L = 1$ and spin $S = 1$ of the two quarks coupled to $J = 0$. Therefore, it has the quantum numbers of the vacuum and, correspondingly, the σ -field entering into the linear σ model (L σ M) has a nonzero vacuum expectation value $\langle 0|\sigma|0\rangle \neq 0$. This leads to a mass m_σ of the σ -meson which is not quantitatively predicted by the L σ M, but quite naturally follows from the quark-level Nambu–Jona-Lasinio model (NJL), by adjusting the predictions of this model to the pion decay constant f_π , the average current-quark mass $m_0 = \frac{1}{2}(m_u + m_d)$ and to the pion mass m_π . In this way parameters can be avoided which otherwise would not be precisely determined. On the other hand, the σ -meson showing up as a broad resonant intermediate state in reactions in which two pions are involved [2] may be understood as a $(u\bar{u} + d\bar{d})/\sqrt{2}$, 1^3P_0 core state in a confining potential, coupled to a $(\pi^+\pi^- - \pi^0\pi^0 + \pi^-\pi^+)/\sqrt{3}$ di-pion state in the continuum, where the two pions are in a relative S -state with

isospin $I = 0$. This coupling lowers the average mass as compared to the confined core state and leads to a broad mass distribution. This dual aspect of the σ -meson leads to two different predictions for the t -channel contribution, $(\alpha - \beta)^t$, of the difference of the electric and magnetic polarizability of the nucleon which have been shown [3] to agree with each other and with the experimental result. In the present work we give details of the derivation of $(\alpha - \beta)^t$ making use of predictions of the NJL model for the σ -meson mass.

2 The quantity $(\alpha - \beta)^t$ predicted from a σ -meson pole

Effective field theories are an excellent tool to adapt properties of QCD to the low-energy regime. In applying these effective field theories to phenomena like the polarizability of the nucleon, care has to be taken to find out what aspects of the phenomenon under consideration can be reproduced and where other theoretical tools are more appropriate.

2.1 Outline of the problem and arguments in favor of the NJL model

In case of Compton scattering [4–10] two types of degrees of freedom (d.o.f.) of the nucleon have to be taken

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into account which may be termed s -channel d.o.f. and t -channel d.o.f. The s -channel d.o.f. are those degrees of freedom of the nucleon which also show up in photoabsorption experiments on the nucleon. The main examples are the nonresonant E_{0+} channel of pion photoproduction corresponding to the “pion cloud”, and the Δ -resonance. The nonresonant E_{0+} channel is of $E1$ multipolarity and contributes only about 40% of the electric polarizability α , in partial contradiction to the frequently stated belief that the “pion cloud” dominates the electric polarizability. The results obtained [3] for the s -channel contributions are $\alpha_p^s = 4.5 \pm 0.5$ and $\alpha_n^s = 5.1 \pm 0.6$ (in units of 10^{-4} fm^3) for the proton and neutron, respectively, to be compared with the corresponding experimental values $\alpha_p^{\text{exp}} = 12.0 \pm 0.6$ and $\alpha_n^{\text{exp}} = 12.5 \pm 1.7$. The Δ -resonance is the origin of the by far dominating part of the electromagnetic polarizabilities. It is of $M1$ multipolarity and, therefore, a strong source of paramagnetic polarizability. Here the numbers are $\beta_p^s = 9.5 \pm 0.5$ and $\beta_n^s = 10.1 \pm 0.6$ to be compared with $\beta_p^{\text{exp}} = 1.9 \pm 0.6$ and $\beta_n^{\text{exp}} = 2.7 \pm 1.8$. Apparently, there exists a strong diamagnetic polarizability which cannot have its origin from the s -channel d.o.f. For illustration, the s -channel d.o.f. are shown in fig. 1. We see the strong $P_{33}(1232)$ (Δ) resonance line and two other prominent lines corresponding to the $D_{13}(1520)$ and the $F_{15}(1680)$ resonances which are sources of $E1$ and $E2$ strength, respectively. The main source of $E1$ strength is due to the nonresonant part of the cross-section, which at the lower energies is due to 1π photoproduction, and due to 2π etc. photoproduction at higher energies.

The additional t -channel d.o.f. are required by Mandelstam analyticity [11]. The invariant amplitudes $A_i(s, t)$ are analytical functions of the two variable s and t and, therefore, the singularities of the t -channel as well as those of the s -channel have to be taken into account. The t -channel d.o.f. may be identified with a (π^0, σ) doublet in the intermediate state where the two mesons are coupled to two photons on the one side and to constituent quarks on the other. In terms of linear polarization the two cases differ from each other by the fact that for the π^0 -meson the directions of linear polarization are perpendicular whereas

for the σ -meson they are parallel. The π^0 -meson corresponds to a pole in the t -plane which contributes the dominant part, γ_π^t , of the backward spin polarizability γ_π . The σ -meson contributes the dominant part, $(\alpha - \beta)^t$, of the difference of electromagnetic polarizabilities $(\alpha - \beta)$. As a broad mass distribution, the σ -meson corresponds to a cut in the t -plane whereas as a particle with a definite mass m_σ it corresponds to a pole in analogy to the π^0 -meson case.

As far as effective field theories are concerned only those versions are of interest which contain the σ -meson explicitly as a particle. This means that the quark-level linear σ model ($L\sigma M$) and the quark-level Nambu–Jona-Lasinio model (NJL) are candidates for representing the t -channel d.o.f. whereas the nonlinear σ model and chiral perturbation theory may be disregarded in connection with the present purpose. The Lagrangian of the $L\sigma M$ consistently describes the mechanism of spontaneous symmetry breaking but does not have the capability of predicting the σ -meson mass m_σ on an absolute scale [12, 13]. This is different in the NJL model [14–21] which describes dynamical symmetry breaking and, thus, predicts a definite σ -meson mass, m_σ , instead of a broad mass distribution. This is the reason why we consider the NJL model as the appropriate candidate for our present study. However, the application of the NJL should be restricted to predictions in connection with the σ -pole contribution. Applications to other aspects of the electromagnetic polarizabilities are not expected to lead to meaningful results.

2.2 The mass m_σ of the σ -meson

The Lagrangian of the NJL model has been formulated in two equivalent ways [15–18]:

$$\mathcal{L}_{\text{NJL}} = \bar{\psi}(i\cancel{\partial} - m_0)\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2], \quad (1)$$

and

$$\mathcal{L}'_{\text{NJL}} = \bar{\psi}i\cancel{\partial}\psi - g\bar{\psi}(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\psi - \frac{1}{2}\delta\mu^2(\sigma^2 + \boldsymbol{\pi}^2) + \frac{gm_0}{G}\sigma, \quad (2)$$

where

$$G = g^2/\delta\mu^2 \quad \text{and} \quad \delta\mu^2 = (m_\sigma^{\text{cl}})^2. \quad (3)$$

Equation (1) describes the four-fermion version of the NJL model and eq. (2) the bosonized version. The quantity ψ denotes the spinor of constituent quarks with two flavors. The quantity G is the coupling constant of the four-fermion version, g the Yukawa coupling constant and $\delta\mu$ a mass parameter entering into the mass counter-term of eq. (2). The coupling constants G , g and the mass parameter $\delta\mu$ are related to each other and to the σ -meson mass in the chiral limit (cl), m_σ^{cl} , as given in (3). The relation $\delta\mu^2 = (m_\sigma^{\text{cl}})^2$ can easily be derived by applying spontaneous symmetry breaking to $\delta\mu$ in analogy to spontaneous symmetry breaking predicted by the $L\sigma M$ for the mass parameter μ entering into this model [13]. In the chiral

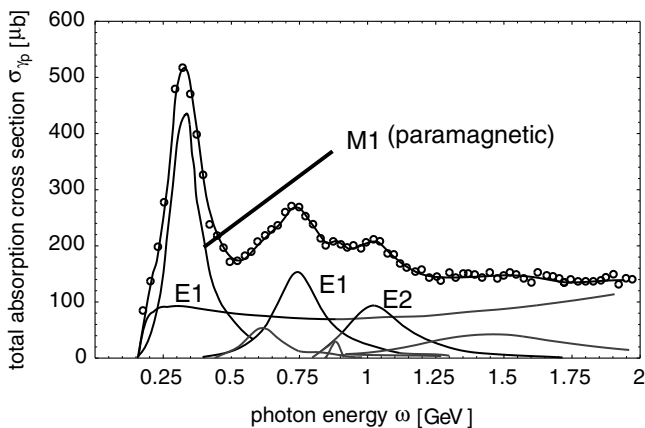


Fig. 1. s -channel d.o.f.: photoabsorption cross-section separated into multipoles.

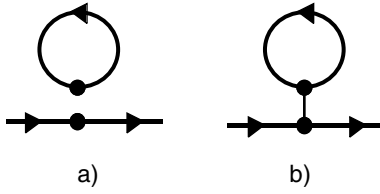


Fig. 2. a) Four-fermion theory tadpole diagram, b) bosonized tadpole diagram [15].

limit ($m_0 \rightarrow 0$) the version of eq. (2) contains the spinor-dependent term in the same way as the $L\sigma M$, whereas the major part of the bosonic (π, σ)-dependent terms of the $L\sigma M$ are absent. This means that with respect to the spinor-dependent terms the $L\sigma M$ and the NJL model are equivalent whereas only a truncated version of the bosonic part of the $L\sigma M$ is produced. The terms in eqs. (1) and (2) containing the average current-quark mass m_0 describe explicit symmetry breaking and can be shown to be equivalent. The insight that the $L\sigma M$ and the NJL model are basically equivalent dates back to the 1960s and 1970s. Figure 2 shows the four-fermion theory tadpole diagram and the bosonized tadpole diagram. The underlying idea has been worked out in detail by Eguchi [16], by Vogl and Weise [17] and Delbourgo and Scadron [22]. Both versions can be exploited to make a prediction for m_σ on an absolute scale. The NJL model faces the problem that use is made of integrals in momentum space which diverge in the infinite momentum limit. To overcome this problem, two different regularization schemes have been developed. The regularization through a cut-off momentum Λ restricts the evaluation of the integrals to the low-momentum region. Dimensional regularization treats the integrals without a cut-off but makes use of the fact that the difference between two diverging integrals is finite.

2.2.1 Four-fermion version of the NJL model with regularization through a cut-off parameter Λ

Using diagrammatic techniques the following equation may be found [18, 19]:

$$M^* = m_0 + 8iGN_c \int^{\Lambda} \frac{d^4p}{(2\pi)^4} \frac{M^*}{p^2 - M^{*2}}, \quad (4)$$

$$f_\pi^2 = -4iN_c M^{*2} \int^{\Lambda} \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - M^{*2})^2}, \quad (5)$$

$$m_\pi^2 = -\frac{m_0}{M^*} \frac{1}{4iGN_c I(m_\pi^2)}, \quad (6)$$

$$I(k^2) = \int^{\Lambda} \frac{d^4p}{(2\pi)^4} \frac{1}{[(p + \frac{1}{2}k)^2 - M^{*2}][(p - \frac{1}{2}k)^2 - M^{*2}]}.$$

The expression given in (4) is the gap equation with M^* being the mass of the constituent quark with the contribution $m_0 = (m_u + m_d)/2$ of the current quarks included and $N_c = 3$ being the number of colors. Equation (5) represents the pion decay constant having the experimental

value $f_\pi = (92.42 \pm 0.26)$ MeV [2]. The expression given in (6) is equivalent to the Gell-Mann–Oakes–Renner relation [23] when formulating (6) in the chiral limit. Using (4)–(6) it is possible to calculate the quantities M^* , f_π and m_π simultaneously and to adjust the parameters G and Λ in such a way that the experimental values for f_π , m_π and m_0 are reproduced. It is apparent that this procedure leaves no room for an unknown parameter so that the predicted value for m_σ is model independent except, of course, for the general use of the theoretical frame provided by the NJL model. Numerical calculations of the type outlined above have been carried out by Hatsuda and Kunihiro [19] applying RPA techniques. The result obtained is

$$m_\sigma \simeq 668 \text{ MeV}, \quad (7)$$

where $m_\sigma \simeq 2M^*$ has been applied.

2.2.2 Bosonized NJL model or dynamical $L\sigma M$ with dimensional regularization

A second way to predict m_σ on an absolute scale introduced by Delbourgo and Scadron [22] is obtained by exploiting eqs. (2) and (3). The starting point [22] are representations of the pion decay constant and the gap equation for the constituent-quark mass, M , in the chiral limit

$$f_\pi^{\text{cl}} = -4iN_c g M \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - M^2)^2}, \quad (8)$$

$$M = -\frac{8iN_c g^2}{(m_\sigma^{\text{cl}})^2} \int \frac{d^4p}{(2\pi)^4} \frac{M}{p^2 - M^2}. \quad (9)$$

The two equations in (6) for the pion mass m_π are not needed because this quantity is equal to zero in the chiral limit, $m_\pi^{\text{cl}} = 0$. Except for a strict application of the chiral limit, these equations differ from those in (4) and (5) by the fact that use has been made of the Goldberger–Treiman relation for the chiral limit:

$$g f_\pi^{\text{cl}} = M \quad (10)$$

in (8) and by replacing G by $-g^2/(m_\sigma^{\text{cl}})^2$ in (9). The different signs in front of the integrals in (4) and (9) follow from the fact that different regularization schemes are applied in the two cases [22].

Then, making use of the identity (dimensional regularization [21, 22])

$$-i \frac{(m_\sigma^{\text{cl}})^2}{16N_c g^2} = \int \frac{d^4p}{(2\pi)^4} \left[\frac{M^2}{(p^2 - M^2)^2} - \frac{1}{p^2 - M^2} \right] = -\frac{iM^2}{(4\pi)^2}, \quad (11)$$

we arrive at

$$(m_\sigma^{\text{cl}})^2 = \frac{N_c g^2 M^2}{\pi^2}, \quad (12)$$

and with $m_\sigma^{\text{cl}} = 2M$ at

$$g = g_{\pi qq} = g_{\sigma qq} = 2\pi/\sqrt{N_c} = 3.63. \quad (13)$$

The σ -meson mass corresponding to this coupling constant is

$$m_\sigma = 666.0 \text{ MeV}, \quad (14)$$

where use has been made of $m_\sigma^2 = (m_\sigma^{\text{cl}})^2 + m_\pi^2$, $f_\pi^{\text{cl}} = 89.8 \text{ MeV}$ [24], $M = 325.8 \text{ MeV}$ and $m_\pi = 138.0 \text{ MeV}$. It is satisfying to note that the results given in (7) and (14) are in an excellent agreement with each other. Since the procedures to arrive at m_σ are quite different in the two cases the good agreement of the two results gives us confidence that the value obtained for m_σ is on a stable basis.

2.3 The two-photon decays of the $f_0(980)$ - the π^0 - and the σ -meson

For mesons P having the constituent-quark structure

$$|q\bar{q}\rangle = \frac{a|u\bar{u}\rangle + b|d\bar{d}\rangle + c|s\bar{s}\rangle}{\sqrt{a^2 + b^2 + c^2}}, \quad (15)$$

the two-photon amplitude is given in the generic form

$$|M(P \rightarrow \gamma\gamma)| = \frac{\alpha_e}{\pi f_P} N_c \frac{a e_u^2 + b e_d^2 + c e_s^2}{\sqrt{a^2 + b^2 + c^2}}, \quad (16)$$

where $\alpha_e = 1/137.04$ and f_P is the decay constant of the meson P (see, e.g., [25, 26]). However, we can use

$$f_P \simeq f_\pi \quad (17)$$

without a major loss of precision. The reason is that f_P does not depend on the flavor wave function of the meson as can be seen in (4). Small deviations from (17) only occur when there is a strange-quark content in the meson as in case of η , η' and $f_0(980)$, because of the larger current-quark mass of the strange quark. This leads us to the following relations:

$$|M(f_0(980) \rightarrow \gamma\gamma)| = \frac{\alpha_e}{\pi f_\pi} N_c \left[\left(-\frac{1}{3} \right)^2 \right] = \frac{1}{3} \frac{\alpha_e}{\pi f_\pi}, \quad (18)$$

$$|M(\sigma \rightarrow \gamma\gamma)| = \frac{\alpha_e}{\pi f_\pi} N_c \left[\left(\frac{2}{3} \right)^2 + \left(-\frac{1}{3} \right)^2 \right] = \frac{5}{3} \frac{\alpha_e}{\pi f_\pi}, \quad (19)$$

$$|M(\pi^0 \rightarrow \gamma\gamma)| = \frac{\alpha_e}{\pi f_\pi} N_c \left[\left(\frac{2}{3} \right)^2 - \left(-\frac{1}{3} \right)^2 \right] = \frac{\alpha_e}{\pi f_\pi}, \quad (20)$$

$$\Gamma_{f_0(980) \rightarrow \gamma\gamma} = \frac{m_{f_0(980)}^3}{64\pi} |M(f_0(980) \rightarrow \gamma\gamma)|^2 = 0.33 \text{ keV}, \quad (21)$$

$$\Gamma_{\sigma \rightarrow \gamma\gamma} = \frac{m_\sigma^3}{64\pi} |M(\sigma \rightarrow \gamma\gamma)|^2 = 2.6 \text{ keV}, \quad (22)$$

$$\Gamma_{\pi^0 \rightarrow \gamma\gamma} = \frac{m_{\pi^0}^3}{64\pi} |M(\pi^0 \rightarrow \gamma\gamma)|^2 = 7.73 \times 10^{-3} \text{ keV}, \quad (23)$$

where $f_P = f_\pi$ is used in all three cases. For the σ -meson we use the ‘‘model independent’’ σ mass, $m_\sigma = 666 \text{ MeV}$, determined in the previous subsection using the method of [22]. The pion decay constant is $f_\pi = (92.43 \pm 0.26) \text{ MeV}$. For the σ -meson we then obtain $\Gamma_{\sigma \rightarrow \gamma\gamma} = 2.6 \text{ keV}$ which

agrees with the experimental result of Boglione and Pennington [27] $\Gamma_{\sigma \rightarrow \gamma\gamma} = (3.8 \pm 1.5) \text{ keV}$ within the errors. In (19) it has been assumed that the coupling of the two photons to the σ -meson proceeds only through its non-strange quark content and that there are no additional contributions due to meson loops as obtained within the framework of the $L\sigma M$ [28, 29]. From a theoretical point of view this neglect of meson-loop contributions is justified through the use of the NJL model where such additional contributions are absent. For sake of completeness, we note that for the π^0 -meson the prediction is $\Gamma_{\pi^0 \rightarrow \gamma\gamma} = 7.73 \times 10^{-3} \text{ keV}$ to be compared with the experimental value $\Gamma_{\pi^0 \rightarrow \gamma\gamma} = (7.74 \pm 0.55) \times 10^{-3} \text{ keV}$ obtained from the mean lifetime $\tau_{\pi^0} = (8.4 \pm 0.6) \times 10^{-17} \text{ s}$ and the branching ratio $\pi^0 \rightarrow \gamma\gamma$ of $(98.798 \pm 0.032)\%$ [2]. We see that for the π^0 -meson as well as for its chiral partner, the σ -meson, the coupling to two photons may be understood as proceeding through their $q\bar{q}$ internal structures. It is remarkable to note that the prediction for the two-photon width of $f_0(980)$ based on a $|\bar{s}s\rangle$ quark structure is in agreement with the experimental value $\Gamma_{f_0(980) \rightarrow \gamma\gamma} = (0.39_{-0.13}^{+0.10}) \text{ keV}$ given by the Particle Data Group (PDG) [2] and also with $\Gamma_{f_0(980) \rightarrow \gamma\gamma} = (0.28_{-0.13}^{+0.09}) \text{ keV}$ given by Boglione and Pennington [27].

There was a longstanding discussion whether or not the $q\bar{q}$ structure of scalar mesons with masses below 1 GeV should be replaced by a $(qq)(\bar{q}\bar{q})$ structure [30]. This alternative appears to be strongly disfavored by the insight that scalar mesons neither correspond to a $q\bar{q}$ structure nor to a $(qq)(\bar{q}\bar{q})$ structure, but to a $q\bar{q}$ structure component coupled to di-meson states [31]. Also, the good agreement of the two-photon decay width of the $f_0(980)$ -meson predicted on the basis of a $|\bar{s}s\rangle$ structure with the corresponding experimental values may be considered as an argument in favor of this structure.

2.4 Polarizabilities and invariant amplitudes

For the discussion of the polarizabilities of the nucleon in terms of Compton scattering the forward direction ($\theta = 0$) and the backward direction ($\theta = \pi$) are of special interest. Denoting the spin-independent and spin-dependent amplitudes for the forward and backward direction by f_0 , g_0 , f_π and g_π , respectively, we arrive at

$$f_0(\omega) = -\frac{\omega^2}{2\pi} [A_3(\nu, t) + A_6(\nu, t)], \quad (24)$$

$$g_0(\omega) = \frac{\omega^3}{2\pi m_N} A_4(\nu, t),$$

$$f_\pi(\omega) = -\frac{\omega\omega'}{2\pi} \left(1 + \frac{\omega\omega'}{m_N^2} \right)^{1/2} \left[A_1(\nu, t) - \frac{t}{4m_N^2} A_5(\nu, t) \right], \quad (25)$$

$$g_\pi(\omega) = -\frac{\omega\omega'}{2\pi m_N} \sqrt{\omega\omega'} \left[A_2(\nu, t) + \left(1 - \frac{t}{4m_N^2} \right) A_5(\nu, t) \right], \quad (26)$$

$$\omega'(\theta = \pi) = \frac{\omega}{1 + 2\frac{\omega}{m_N}}, \quad \nu = \frac{1}{2}(\omega + \omega'), \quad (27)$$

$$t(\theta = 0) = 0, \quad t(\theta = \pi) = -4\omega\omega',$$

where A_i are the invariant amplitudes in the standard definition [4–10] and m_N the nucleon mass. For the electric, α , and magnetic, β , polarizabilities and the spin polarizabilities γ_0 and γ_π for the forward and backward directions, respectively, we obtain the relations

$$\begin{aligned} \alpha + \beta &= -\frac{1}{2\pi} [A_3^{\text{nB}}(0, 0) + A_6^{\text{nB}}(0, 0)], \\ \alpha - \beta &= -\frac{1}{2\pi} [A_1^{\text{nB}}(0, 0)], \\ \gamma_0 &= \frac{1}{2\pi m_N} [A_4^{\text{nB}}(0, 0)], \\ \gamma_\pi &= -\frac{1}{2\pi m_N} [A_2^{\text{nB}}(0, 0) + A_5^{\text{nB}}(0, 0)], \end{aligned} \quad (28)$$

where A_i^{nB} are the non-Born parts of the invariant amplitudes.

According to eqs. (24) to (28) the following linear combinations of invariant amplitudes are of special importance because they contain the physics of the four fundamental sum rules, *viz.* the BEFT [32] sum rule for $(\alpha - \beta)$, the LN [33] sum rule for γ_π , the BL [34] sum rule for $(\alpha + \beta)$, and the GDH [35] sum rule for the square of the anomalous magnetic moment κ^2 , respectively:

$$\tilde{A}_1(\nu, t) \equiv A_1(\nu, t) - \frac{t}{4m_N^2} A_5(\nu, t), \quad (29)$$

$$\tilde{A}_2(\nu, t) \equiv A_2(\nu, t) + \left(1 - \frac{t}{4m_N^2}\right) A_5(\nu, t), \quad (30)$$

$$\tilde{A}_3(\nu, t) \equiv A_{3+6}(\nu, t) \equiv A_3(\nu, t) + A_6(\nu, t), \quad (31)$$

$$\tilde{A}_4(\nu, t) \equiv A_4(\nu, t). \quad (32)$$

Therefore, these linear combinations of invariant amplitudes may be considered as generalized polarizabilities containing the essential physics of the polarizability of the nucleon.

In the following use is made of the relations concerning $\alpha + \beta$, $\alpha - \beta$ and γ_π , whereas the relation concerning γ_0 is written down only for the sake of completeness.

2.5 Fixed- θ dispersion relations at $\theta = \pi$

Fixed- θ dispersion relations have the advantage that a clear-cut separation of the s -channel and the t -channel is possible, *i.e.* there definitely is no double counting of empirical input [11]. In the following, we are interested in the backward spin-polarizability γ_π and the difference of electromagnetic polarizabilities $\alpha - \beta$ which have a firm relation to the invariant amplitudes at $\theta = \pi$, where one can write down [7, 36] a dispersion integral as

$$\begin{aligned} \text{Re } A_i(s, t) &= A_i^{\text{B}}(s, t) + A_i^{t\text{-pole}}(s, t) \\ &+ \frac{1}{\pi} \int_{(m_N + m_\pi)^2}^{\infty} ds' \text{Im}_s A_i(s', \tilde{t}) \left[\frac{1}{s' - s} + \frac{1}{s' - u} - \frac{1}{s'} \right] \\ &+ \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} dt' \frac{\text{Im}_t A_i(\tilde{s}, t')}{t' - t}, \end{aligned} \quad (33)$$

where $\text{Im}_s A(s', \tilde{t})$ is evaluated along the hyperbola given by

$$s' u' = m_N^4, \quad s' + \tilde{t} + u' = 2m_N^2, \quad (34)$$

and $\text{Im}_t A_i(\tilde{s}, t')$ runs along the path defined by the hyperbola

$$\tilde{s} \tilde{u} = m_N^4, \quad \tilde{s} + t' + \tilde{u} = 2m_N^2. \quad (35)$$

The amplitude $A_i^{t\text{-pole}}(s, t)$ entering into (33) describes the contribution of t -channel poles to the scattering amplitudes which for pseudoscalar mesons may be written in the form

$$A_2^{\pi^0 + \eta + \eta'}(t) = \frac{g_{\pi NN} F_{\pi^0 \gamma \gamma} \tau_3}{t - m_{\pi^0}^2} + \frac{g_{\eta NN} F_{\eta \gamma \gamma}}{t - m_\eta^2} + \frac{g_{\eta' NN} F_{\eta' \gamma \gamma}}{t - m_{\eta'}^2}, \quad (36)$$

where the quantities $g_{\pi NN}$, etc. are the meson-nucleon coupling constants and the quantities $F_{\pi^0 \gamma \gamma}$, etc. the two-photon decay amplitudes. The last term in (33) represents the contribution of t -channel cuts to the scattering amplitudes which later on will be discussed in connection with the scalar-isoscalar t -channel. From (36) the following relation for the t -channel part of the backward spin-polarizability may be obtained:

$$\gamma_\pi^t = \frac{1}{2\pi m_N} \left[\frac{g_{\pi NN} F_{\pi^0 \gamma \gamma} \tau_3}{m_{\pi^0}^2} + \frac{g_{\eta NN} F_{\eta \gamma \gamma}}{m_\eta^2} + \frac{g_{\eta' NN} F_{\eta' \gamma \gamma}}{m_{\eta'}^2} \right]. \quad (37)$$

The pion-nucleon coupling constant is given by the experimental value $g_{\pi NN} = 13.169 \pm 0.057$ [37]. The corresponding quantities for the η - and η' -meson cannot be much different. The reason is that on a quark level and in the chiral limit the coupling constant $g_{\pi qq}$ shows up as a universal proportionality constant between the quantities f_π^{cl} and M given in (8) and (9), respectively. A different problem is the sign of the two-photon decay amplitude where we have

$$\begin{aligned} F_{\pi^0 \gamma \gamma} &= -|M(\pi^0 \rightarrow \gamma \gamma)|, \\ F_{\eta \gamma \gamma} &= \pm |M(\eta \rightarrow \gamma \gamma)|, \\ F_{\eta' \gamma \gamma} &= \pm |M(\eta' \rightarrow \gamma \gamma)|. \end{aligned} \quad (38)$$

The minus sign in case of $F_{\pi^0 \gamma \gamma}$ is well established [5, 33, 38] whereas the other signs are less well known.

The expression (37) for the t -channel part of the backward spin-polarizability has been tested and found valid. Details may be found in table 1. In [33] it is proposed to adopt the minus sign for the η and η' contributions in eq. (38). By comparing line 4 with lines 6 and 7 in table 1 we see that a better agreement with the experimental values is obtained when the plus sign is used.

For $(\alpha - \beta)$ we have the choice to either use the pole representation as appropriate for the σ -meson as entering into the NJL model or the cut representation as appropriate for the σ -meson as a broad resonant intermediate state.

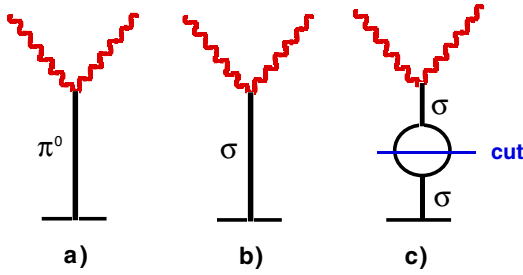
Here we first discuss the pole representation (see fig. 3b)). Then, the σ -meson may be understood as a $(u\bar{u} + d\bar{d})/\sqrt{2}$ configuration having a definite mass m_σ

Table 1. Backward spin-polarizability for the proton and the neutron (units 10^{-4} fm^4).

1	Spin-pol.	Proton	Neutron	
2	γ_π	-38.7 ± 1.8	$+58.6 \pm 4.0$	Experiment [10]
3	γ_π^s	$+7.1 \pm 1.8$	$+9.1 \pm 1.8$	Sum rule [33]
4	γ_π^t	-45.8 ± 2.5	$+49.5 \pm 4.4$	line 2 – line 3
5	γ_π^t	-46.7	$+46.7$	π^0 -pole only
6	γ_π^t	-45.1	$+48.3$	$\pi^0 + \eta + \eta'$ -poles ^{a)}
7	γ_π^t	-48.3	$+45.1$	$\pi^0 + \eta + \eta'$ -poles ^{b)}

a) η and η' contributions assumed to be positive numbers (line 6).

b) η and η' contributions assumed to be negative numbers (line 7).

**Fig. 3.** a) π^0 -pole diagram. b) σ -pole diagram, c) scalar-isoscalar t -channel as entering into the BEFT sum rule.

as predicted by the quark-level NJL model. The corresponding amplitude is constructed in analogy to the pseudoscalar pole and given by

$$A_1^{t\text{-pole}}(t) = \frac{g_{\sigma NN} F_{\sigma\gamma\gamma}}{t - m_\sigma^2}, \quad (39)$$

with $g_{\sigma NN}$ being the σ -nucleon coupling constant, $F_{\sigma\gamma\gamma}$ the two-photon σ decay amplitude and m_σ the σ mass¹. Then some consideration shows that the t -channel part of the polarizability difference is given by

$$\begin{aligned} (\alpha - \beta)_{p,n}^t &= \frac{g_{\pi NN} F_{\sigma\gamma\gamma}}{2\pi m_\sigma^2} = \frac{5\alpha_e g_{\pi NN}}{6\pi^2 m_\sigma^2 f_\pi} \\ &= 15.2 \approx \frac{5\alpha_e g_{\pi NN}^2}{6\pi^2 m_\sigma^2 g_A m_N}, \end{aligned} \quad (40)$$

in units of 10^{-4} fm^3 , where $\alpha_e = 1/137.04$, $g_{\sigma NN} \equiv g_{\pi NN} = 13.169 \pm 0.057$ [37], $f_\pi = (92.42 \pm 0.26) \text{ MeV}$, $m_\sigma = 666 \text{ MeV}$ as derived in sect. 2.2 using the method of [22]. In (40) the quark-model prediction $F_{\sigma\gamma\gamma} = +|M(\sigma \rightarrow \gamma\gamma)|$ with $N_c = 3$ has been used. The identity $g_{\sigma NN} \equiv g_{\pi NN}$ is predicted by the L σ M and has been experimentally confirmed by Durso *et al.* [39]. On the r.h.s. of eq. (40) use has been made of the approximately valid Goldberger-Treiman relation, where $g_A = 1.255 \pm 0.006$ is the axial vector coupling constant.

¹ Occasionally it has been proposed to modify the pole amplitude given in (39) by considering the quantities $g_{\sigma NN}$ and $F_{\sigma\gamma\gamma}$ as t -dependent formfactors. Such a procedure, however, is not allowed because it is incompatible with the requirements of dispersion theory.

An upper limit for the possible correction to $(\alpha - \beta)_{p,n}^t$ as given in (40) due to the $f_0(980)$ -meson can be calculated making the assumption that the coupling constants $g_{f_0(980)NN}$ and $g_{\pi NN}$ are equal to each other. The result obtained is a possible correction of not more than 9%.

3 The quantity $(\alpha - \beta)^t$ predicted from the reactions $\pi\pi \rightarrow \sigma \rightarrow \pi\pi$, $\gamma\gamma \rightarrow \sigma \rightarrow \pi\pi$ and $N\bar{N} \rightarrow \sigma \rightarrow \pi\pi$

A different approach to a calculation of $(\alpha - \beta)^t$ which only makes use of experimental data without specific assumptions about the internal structure of the σ -meson is provided by the BEFT sum rule [32]. Instead of exploiting eq. (40), the following relation is considered:

$$\gamma\gamma \rightarrow |\sigma\rangle \rightarrow \pi\pi \rightarrow |\sigma\rangle \rightarrow N\bar{N}, \quad (41)$$

and replaced by

$$\gamma\gamma \rightarrow |\sigma\rangle \rightarrow \pi\pi, \quad (42)$$

$$\pi\pi \rightarrow |\sigma\rangle \rightarrow N\bar{N}, \quad (43)$$

obtained by means of a t -channel cut (see fig. 3c)). The further procedure is to use the unitarity relation

$$\begin{aligned} \text{Im}_t T(\gamma\gamma \rightarrow N\bar{N}) &= \frac{1}{2} \sum_n (2\pi)^4 \delta^4(P_n - P_i) \\ &\times T(\gamma\gamma \rightarrow n) T^*(N\bar{N} \rightarrow n), \end{aligned} \quad (44)$$

where the sum on the r.h.s. is taken over all allowed intermediate states n having the same total 4-momentum as the initial state. Furthermore, $\pi\pi$ intermediate states are taken into account where the spin is $J = 0$ and the isospin $I = 0$. These are the quantum numbers of the intermediate σ -meson.

If we restrict ourselves in the calculation of the t -channel absorptive part to intermediate states with two pions with angular momentum $J \leq 2$, the BEFT sum rule [32] gets a convenient form for calculations:

$$\begin{aligned} (\alpha - \beta)_{p,n}^t &= \frac{1}{16\pi^2} \int_{4m_\pi^2}^{\infty} \frac{dt}{t^2} \frac{16}{4m_N^2 - t} \left(\frac{t - 4m_\pi^2}{t} \right)^{1/2} \\ &\times \left[f_+^0(t) F_0^{0*}(t) - \left(m_N^2 - \frac{t}{4} \right) \left(\frac{t}{4} - m_\pi^2 \right) f_+^2(t) F_0^{2*}(t) \right], \end{aligned} \quad (45)$$

where $f_+^{(0,2)}(t)$ and $F_0^{(0,2)}(t)$ are the partial-wave helicity amplitudes of the processes $N\bar{N} \rightarrow \pi\pi$ and $\pi\pi \rightarrow \gamma\gamma$ with angular momentum $J = 0$ and 2, respectively, and isospin $I = 0$.

Except for the quantum numbers, properties of the σ -meson enter through the amplitudes $f_+^{(0,2)}(t)$ and $F_0^{(0,2)}(t)$ corresponding to the reactions (42) and (43). These amplitudes incorporate the phase $\delta_J^I(s) = \delta_0^0(s)$ which is extracted from π - π scattering data as obtained in $\pi N \rightarrow$

Table 2. Summary on electromagnetic polarizabilities in units of 10^{-4} fm^3 .

1		Proton	Neutron
2	BL sum rule	$(\alpha + \beta)_p = 13.9 \pm 0.3$	$(\alpha + \beta)_n = 15.2 \pm 0.5$
3	Compton scattering	$(\alpha - \beta)_p = 10.1 \pm 0.9$	$(\alpha - \beta)_n = 9.8 \pm 2.5$
4	BEFT sum rule	$(\alpha - \beta)_p^s = -5.0 \pm 1.0$	$(\alpha - \beta)_n^s = -5.0 \pm 1.0$
5	Line 3 – line 4	$(\alpha - \beta)_p^t = 15.1 \pm 1.3$	$(\alpha - \beta)_n^t = 14.8 \pm 2.7$
6	Experiment	$\alpha_p = 12.0 \pm 0.6$	$\alpha_n = 12.5 \pm 1.7$
7	s -channel only	$\alpha_p^s = 4.5 \pm 0.5$	$\alpha_n^s = 5.1 \pm 0.6$
8	t -channel only	$\alpha_p^t = 7.5 \pm 0.8$	$\alpha_n^t = 7.4 \pm 1.8$
9	Experiment	$\beta_p = 1.9 \mp 0.6$	$\beta_n = 2.7 \mp 1.8$
10	s -channel only	$\beta_p^s = 9.5 \pm 0.5$	$\beta_n^s = 10.1 \pm 0.6$
11	t -channel only	$\beta_p^t = -7.6 \pm 0.8$	$\beta_n^t = -7.4 \pm 1.9$

$N\pi\pi$ scattering experiments. The information contained in the phase $\delta_0^0(s)$ can be understood as being due to a resonance with the pole parameters [2, 40, 41]

$$\sqrt{s^{\text{pole}}} \approx (500 - i 250) \text{ MeV} \quad (46)$$

and the special property of a 90° crossing of the phase at

$$\sqrt{s}(\delta_0^0 = 90^\circ) \approx 900 \text{ MeV}. \quad (47)$$

The apparent mismatch of the 90° phase crossing expected for the resonance part as given by (46) and the observed 90° phase crossing as given by (47) has led to numerous considerations [42–44] among which the possible existence of a background [42] or a second pole at negative mass parameter m^2 [43] or one pole with an s -dependent width $\Gamma(s)$ [44] played a role. This discussion shows that a generally accepted model for the σ -meson as a resonant state is still missing (see, however, the last entry in [31] and references therein). Furthermore, the inclusion of the $\pi\pi$ phase relation into the $\gamma\gamma \rightarrow \pi\pi$ and $\bar{N}N \rightarrow \pi\pi$ amplitudes is not an easy task. This may be the reason why only recently some consistency in the prediction of $(\alpha - \beta)^t$ from the BEFT sum rule [32] has been obtained. This has led to

$$(\alpha - \beta)_{p,n}^t = (14.0 \pm 2.0) \quad (\text{ref. [10]}), \quad 16.46 \quad (\text{ref. [7]}), \quad (48)$$

where the first value has been obtained by Levchuk *et al.* (see [10]) and the second value by Drechsel *et al.* [7]. The large error given in ref. [10] takes care of the uncertainties contained in the appropriate choice of experimental data. The arithmetic average of the two results is $(\alpha - \beta)_{p,n}^t = 15.3 \pm 1.3$. This number has to be compared with the experimental data $(\alpha - \beta)_p^t = 15.1 \pm 1.3$, $(\alpha - \beta)_n^t = 14.8 \pm 2.7$ (see ref. [10]). We see that the predictions for $(\alpha - \beta)^t$ obtained from the σ -meson pole based on the NJL model and from the BEFT sum rule lead to agreement with each other and to agreement with experiment.

4 Discussion

According to our recent analysis [3, 9, 10] the experimental polarizabilities may be summarized in the form given in

table 2. The quantities $\alpha_p, \beta_p, \alpha_n, \beta_n$ are the experimental electric and magnetic polarizabilities for the proton and neutron, respectively. The quantities with an upper label s are the corresponding electric and magnetic polarizabilities where only the s -channel degrees of freedom are included. These latter quantities have been obtained by making use of the fact that $(\alpha + \beta)$, when calculated from forward-angle dispersion theory, as given by the Baldin or Baldin-Lapidus (BL) sum rule [34]

$$(\alpha + \beta) = \frac{1}{2\pi^2} \int_{m_\pi + \frac{m_\pi}{2m_N}}^\infty \frac{\sigma_{\text{tot}}(\omega)}{\omega^2} d\omega, \quad (49)$$

has no t -channel contribution, *i.e.* $(\alpha + \beta) = (\alpha + \beta)^s$, and by using the estimate $(\alpha - \beta)_{p,n}^s = -5.0 \pm 1.0$ obtained from the s -channel part of the BEFT sum rule [32]

$$(\alpha - \beta)^s = \frac{1}{2\pi^2} \int_{m_\pi + \frac{m_\pi}{2m_N}}^\infty \sqrt{1 + \frac{2\omega}{m_N}} [\sigma(\omega, E1, M2, E3, \dots) - \sigma(\omega, M1, E2, M3, \dots)] \frac{d\omega}{\omega^2} \quad (50)$$

both for the proton and the neutron (see [10]). The absence of a t -channel contribution to $(\alpha + \beta)$ follows from the observation [10] that the BL sum rule is fulfilled. This observation is explained by the fact that in the forward direction vector-meson dominance converts the t -channel contribution into the Regge part of the photoabsorption cross-section. The numbers in line 5 of table 2 are the t -channel contributions to $(\alpha - \beta)$ obtained from the experimental values $(\alpha - \beta)_p = 10.1 \pm 0.9$ and $(\alpha - \beta)_n = 9.8 \pm 2.5$ and the estimate for $(\alpha - \beta)_{p,n}^s$. As noted before, we see that the experimental values for α are much larger than the s -channel contributions alone, whereas for the magnetic polarizabilities the opposite is true. For the magnetic polarizability it makes sense to identify the large difference between the experimental value and the s -channel contribution with the diamagnetic polarizability. This means that we may consider β^t as the diamagnetic polarizability.

Certainly, by identifying the expression obtained for $(\alpha - \beta)_{p,n}^t$ in eq. (40) with that of eq. (45) a sum rule is obtained. This finding is also supported by the two graphs b) and c) in fig. 3. Furthermore, we see in table 3 that

Table 3. Difference of electromagnetic polarizabilities $(\alpha - \beta)_{p,n}^t$ in the t -channel (in units of 10^{-4} fm^3). The result given for the σ -meson cut or BEFT sum rule is the arithmetic average of the results of Drechsel *et al.* [7] and Levchuk *et al.* (see [3, 10]).

	$(\alpha - \beta)_p^t$	$(\alpha - \beta)_n^t$
Experiment	15.1 ± 1.3	14.8 ± 2.7
σ -pole	15.2	15.2
BEFT sum rule	15.3 ± 1.3	15.3 ± 1.3

the experimental results obtained for $(\alpha - \beta)^t$ from the experiments, from the σ -meson pole and from the σ -meson cut agree with each other and thus give also strong support for the existence and validity of the sum rule. We expect that by studying this sum rule in more detail some more insight into the structure of the σ -meson is obtained. In such a study the role of the quark-level NJL model would be to describe the σ -meson as the particle of the σ field with a definite mass $m_\sigma = 666 \text{ MeV}$, whereas the BEFT sum rule exploits on-shell properties of the σ -meson as there is, *e.g.*, the phase relation $\delta_0^0(s)$. The sum rule we are proposing has the property of linking on-shell aspects of the σ -meson with properties of the σ -meson as the particle of the σ field.

As a concluding remark we wish to state that the present paper closes a circle, starting with the work of Hearn and Leader (1962) [11] where the role of the t -channel contributions to the Compton scattering amplitudes has been clarified. In the present work we have shown that the (σ, π^0) particle doublet is capable of quantitatively reproducing this t -channel contribution, with minor additional contributions from the $f_0(980)$, the η - and the η' -meson. Since the t -channel contribution cannot be interpreted in terms of nucleon resonances or in terms of the meson cloud as viewed in photon-meson production experiments, it is reasonable to interpret this contribution in terms of a short-range property of the constituent quarks, as proposed in [10]. More explicitly this means that each constituent quark $|q\rangle$ of the nucleon is converted into a $|(\sigma, \pi^0)q\rangle$ t -channel resonant intermediate state during the Compton scattering process. However, this resonant intermediate state is located in the unphysical region at positive t . At $\theta = \pi$ the π^0 -meson is involved in those scattering processes where the incoming and the outgoing photon have perpendicular directions of linear polarization whereas for the σ -meson the directions of linear polarization are parallel.

Other approaches using model calculations (see [10] for a summary) or diagrammatic techniques [45–47] do not take care of the scalar-isoscalar t -channel contribution in a sufficient way. This means that they are important with respect to a test of the underlying theoretical ansatz, but they cannot be considered as a quantitative descriptions of the electromagnetic polarizabilities of the nucleon. It is very interesting to analyze the different strategies contained in these approaches [45–47] to cope with the serious problem caused by the missing scalar-isoscalar t -channel

in the light of dispersion theory. The most transparent of these strategies are the neglect of the contribution of the Δ -resonance [45] and the introduction of an empirical counter term of unnatural size [46].

The strategy of neglecting the Δ -resonance contribution has been analysed by L'vov [48] in terms of dispersion theory. It has been confirmed that the simultaneous neglect of the scalar-isoscalar t -channel and of the Δ -resonance leads to an approximate agreement with the experimental data. Furthermore, it has been confirmed that the use of the heavy-baryon approximation ($m_N \rightarrow \infty$) leads to an improvement of the agreement with the experimental data, though no good physical reason has been found for such a replacement. Finally, it has been shown [48] that the evaluation of chiral loops [45] leads to an approximate agreement with the contributions of the E_{0+} component to α and β . However, this is only the case when the empirical CGLN amplitude E_{0+} is replaced by the Born approximation E_{0+}^{Born} .

Since there is no definite interpretation available for the empirical counter terms of unnatural sizes [46], $\delta\alpha$ and $\delta\beta$, it may be allowed to tentatively compare these terms with the present t -channel polarizabilities $\alpha_{p,n}^t$ and $\beta_{p,n}^t$ contained in table 2. Qualitatively, these terms $\delta\alpha$ and $\delta\beta$ are introduced to produce diamagnetism. Furthermore, their physical nature is assumed to be related to short-distance phenomena of some kind. These two properties suggest that $\delta\alpha$ and $\delta\beta$, on the one hand, and $\alpha_{p,n}^t$ and $\beta_{p,n}^t$, on the other, should have some common features or even may be identical. Unfortunately, the numbers obtained empirically, *i.e.* $\delta\alpha = -5.92 \pm 1.36$ and $\delta\beta = -10.68 \pm 1.17$, are not in a good agreement with the numbers obtained for $\alpha_{p,n}^t = -\beta_{p,n}^t = 7.6$.

5 Summary

The present paper succeeds in a quantitative derivation of the σ -meson pole contribution to $(\alpha - \beta)^t$ which formerly was introduced and treated semi-quantitatively by adjusting to experimental data [5]. Furthermore, there was no strict argument for the pole structure of this contribution, since the only knowledge about the scalar-isoscalar t -channel was obtained from the BEFT sum rule which makes use of a t -channel cut and not of a t -channel pole. In the present paper we show that all the relevant parameters of the σ -meson pole, *viz.* the σ -meson mass m_σ and properties of the two-photon decay width $\Gamma_{\gamma\gamma}$ follow from the NJL model without any adjustable parameter. Making use of previous work [19, 22], the mass m_σ is obtained in two different versions of the NJL model and two different regularization schemes. In the first case [19] the four-fermion version of the NJL model is used and the regularization scheme makes use of a cut-off parameter Λ . Both quantities, the cut-off parameter Λ and the coupling G are obtained within this regularization scheme by adjusting to empirical data as there are the pion decay constant f_π , the pion mass m_π and the current-quark mass m_0 . The result is $m_\sigma = 668 \text{ MeV}$. In the second case [22], the bosonized

version of the NJL model is applied and the regularization scheme makes use of dimensional scaling, *i.e.* no cut-off parameter Λ is present so that the integrals extend to infinity. Instead, use is made of the fact that the difference of the diverging integrals for the constituent-quark mass M and the pion decay constant f_π is finite. The result is $m_\sigma = 666$ MeV. It is remarkable and has not been pointed out before that these two completely different treatments of the NJL model perfectly lead to the same result. This gives us confidence that conclusions based on this number for the σ -meson mass are on a sound basis. In case of the two-photon decay width $\Gamma_{\gamma\gamma}$ the NJL model is used to justify the neglect of couplings of the σ -meson to two photons via meson loops. Such additional couplings follow from the $L\sigma M$. If taken into account, the additional couplings would partly destroy the good agreement between theory and experiment. Therefore, the argument delivered by the NJL model is important.

A further new result obtained is the calculation of the two-photon decay width $\Gamma_{\gamma\gamma}$ of the $f_0(980)$ -meson. This calculation is based on a $s\bar{s}$ structure of the $f_0(980)$ -meson and leads to agreement with experimental results. We consider this as an argument that indeed the $f_0(980)$ -meson and the σ -meson have a $q\bar{q}$ core and not a $qq\bar{q}\bar{q}$ structure as suggested in other approaches [30]. This result is important for the present investigation as well as for the physics of scalar mesons in general. In addition, for the first time the contribution of the $f_0(980)$ -meson to $(\alpha - \beta)^t$ has been calculated. A further new result are arguments in favor of positive decay amplitudes $F_{\eta\gamma\gamma}$ and $F_{\eta'\gamma\gamma}$ which formerly were believed to be negative.

Via the two representations of $(\alpha - \beta)^t$ a quantitative link is obtained between the σ -meson as the particle of the σ field and the σ -meson as showing up as an extremely short-lived intermediate state in reactions where two pions are involved. This observation may be exploited to get insight into the structure of the σ -meson by constructing a model which makes the relation between the graphs b) and c) in fig. 3 transparent.

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